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MULTIPLE MIDCOURSE MANEUVERS IN INTERPLANETARY GUIDANCE

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SUMMARY

An attempt has been made to investigate the number and location of midcourse maneuvers in an interplanetary mission.

The case of a fly-by Jovian mission with a spin stabilized spacecraft is considered. The mission objectives are a fly-by within 10^6 km (6 diameters) of the planet Jupiter and a flight time from earth to Jupiter of 500 days.

The spacing-ratio method has been applied and it is shown that a constant spacing-ratio of 2.5 for the velocity-sensitivity leads to a near optimum spacing of correction times.

MULTIPLE MIDCOURSE MANEUVERS IN INTERPLANETARY GUIDANCE

INTRODUCTION

A spacecraft moving in free flight toward some target region in space is guided to its final destination by applying one or more small velocity impulse corrections at certain times along the trajectory to null the predicted target error. The estimate of the target error is achieved by an orbit determination process; the required corrections are computed using linear perturbation theory, and the impulse is delivered by a rocket motor, which applies an acceleration to the spacecraft for a relatively short period of time. The selection of times for performing the velocity corrections to the actual trajectory, and the determination of what fraction of the predicted target errors is to be nulled by each maneuver is termed by guidance law or guidance policy.

The technique for applying a single-impulse velocity correction to the trajectory of a spacecraft has been developed at Jet Propulsion Laboratory and is now almost classic. This technique has led to the Mariner II fly-by mission to the planet Venus and also to the Mariner IV fly-by mission to the planet Mars. In this case, a suitable single-maneuver time is chosen from preflight studies of orbit determination and execution error statistics, and the correction capability to be carried aboard the spacecraft is determinated by mapping the covariance matrix of injection guidance errors to the selected maneuver point [1] to obtain the covariance matrix of "velocity-to-be-gained" components.

The consideration and realization of certain flights with more rigid mission objectives and higher accuracies such as Apollo mission, satellite injection about planets and soft landing, led us to believe that a single maneuver is inadequate to accomplish the mission objectives with the required degree of accuracy.

The accuracy that can be obtained from multiple midcourse corrections is a considerable improvement over the single maneuver. On the other hand, the situation becomes much more complex when more than one maneuver is considered, because the future guidance and tracking policy must be considered for the application of a maneuver at any given time.

The target error criterion and the observed accuracy must be defined, as well as the bound on the total velocity correction that can be applied. This question has been investigated, as an optimization problem by Battin [2], Breakwell [3], Striebel and Breakwell [4], Lawden [5], Pfeiffer [1], and others.

In general, there would be a sequence of times at which impulsive corrections might be made. Assuming that the task of the guidance system is to null the predicted target error, and that orbit determination data to be taken are specified, the sequence of times for performing the corrections is associated with the guidance law. The selection of these times is an example of an optimal stochastic control problem.

To arrive at criteria for the selection of these times, several analytical methods have been proposed. The main techniques developed are the three following schemes for the determination of the guidance law.

1. The variance ratio method, suggested by Battin [2], in which a criterion for the timing of corrective impulses has been proposed. This criterion is based on the ratio of the required velocity correction to the uncertainty in estimated miss distance, assuming that each maneuver nulls out the estimated miss distance error. This criterion is not directly related to a concept of minimum fuel expenditure.

2. The spacing ratio method, developed by Breakwell [3], in which one seeks the timing of corrective impulses to minimize the average total maneuvers under the assumption that:

   a. Each maneuver nulls out the estimated miss distance error.

   b. The error in estimating miss distance is due entirely to an error in estimating the instantaneous velocity vector. The optimum choice for the spacing of corrections is one which minimizes the total velocity correction or, equivalently, minimizes the total required fuel.

3. The minimum error method, proposed by Pfeiffer [1], in which the problem is approached from a dynamic programming point of view in order to formulate an adaptive policy that seeks to minimize the mean-squared target error, subject to a propellant constraint. In this method the maneuver times t_i are designated, and the possibility of performing a maneuver at each of these times is allowed.

The purpose of this paper is to investigate the number and location of the correction points on a fly-by Jovian mission. The choice of the spacing-ratio method is mainly due to its characteristic features.

The symbols and nomenclature used in this paper are taken from reference [6] which are commonly used by the Mission Analysis Office. The nominal trajectory used for this Galactic Probe is adopted from [7] with the same elements and characteristics.

STATEMENT AND ANALYSIS OF THE PROBLEM

Suppose a spacecraft moves in free flight on its way from earth to the planet Jupiter, so that its trajectory would pass within 10^6 km of the planet Jupiter. The spacecraft attitude will be restricted during its motion, and more specifically its attitude control law will be assumed that its spin axis $\vec{A}^\circ$ (unit vector) inertially fixed as parallel to the injection velocity $\overrightarrow{\Delta V}$, Figure 1, (reference 8), that is to say:

$$\vec{A}^\circ = \overrightarrow{\Delta V}^\circ, \tag{1}$$

where

$$\overrightarrow{\Delta V}^\circ = \frac{\overrightarrow{\Delta V}}{|\overrightarrow{\Delta V}|} .$$

In fact, if proper precautions are taken to prevent decay of spin rate, spin stabilization has the most general application of all passive systems.

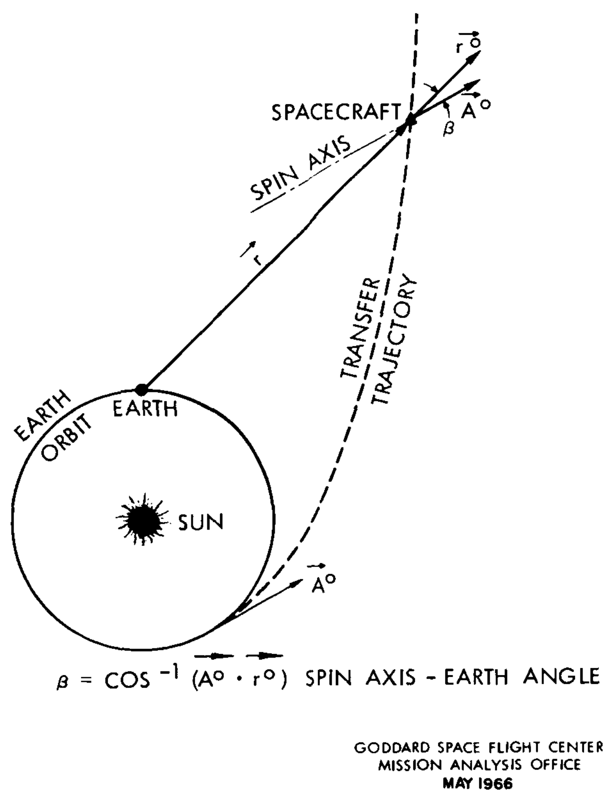

Figure 1—Jupiter Transfer Geometry

Midcourse maneuvers are also restricted to be applied along the spin axis $\vec{A}^{\circ}$.

Except in the immediate vicinity of the spheres of influence of the earth and the planet Jupiter, the nominal trajectory of the spacecraft is essentially a heliocentric ellipse. This problem is three-dimensional, but we assume the orbits of earth and Jupiter and transfer ellipse are coplanar, and the treatment is two-dimensional.

The actual transfer trajectory is somewhat different from the nominal trajectory due to injection errors and therefore the spacecraft would miss the desired terminal target by a miss distance $B(t_{n-1})$ if no corrective action were taken.

It is desired to apply at time t_n a corrective velocity impulse ΔV_n at junction point C_n, Figure 2, to reduce the error at the terminal point.

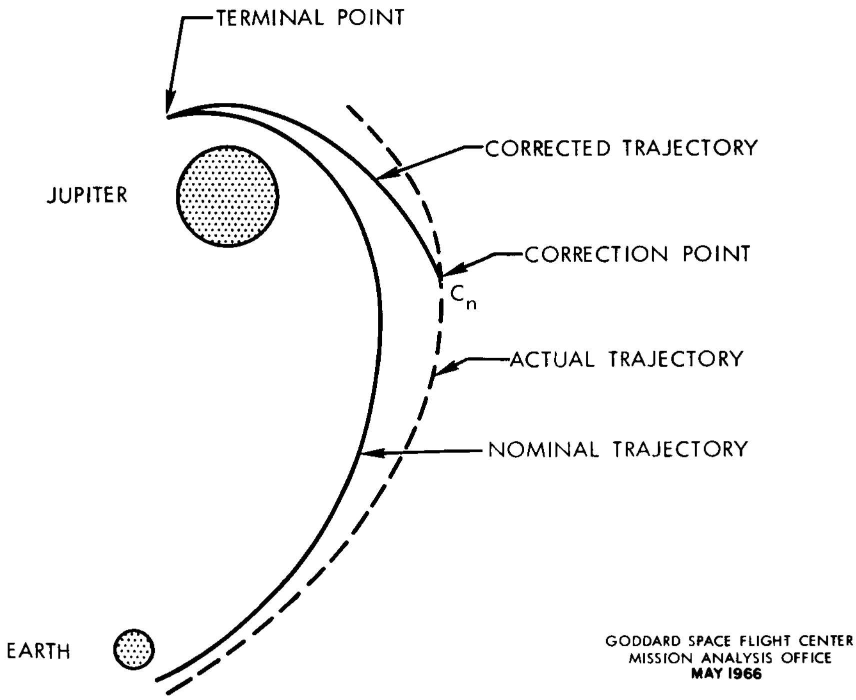

Figure 2–Interplanetary Trajectory

By applying a linearized error theory one finds [3][1] the expression of the computed corrective velocity impulse $\overline{\Delta V_n}$ as follows:

$$\left(\frac{\partial \overrightarrow{B(t_n)}}{\partial V}\right) \cdot \left(\overrightarrow{\overline{\Delta V_n}}\right) + \overline{B(t_{n-1})} = 0, \tag{2}$$

where $\overline{B(t_{n-1})}$ denotes an estimate of the miss distance $B(t_{n-1})$ prior to the nth impulse and subsequent to the $(n-1)^{th}$ impulse. The partial derivative is to be evaluated along the nominal trajectory on which there are no impulses.

It is convenient to use the subscript n and set, in the sequel,

$$B(t_n) = B_n .$$

The miss distance B_{n-1} is not usually correctly estimated, that is to say, a correction at the junction point C_n still leaves us with a miss distance B_n which may be reduced by further corrections. If the estimated $\overline{B_{n-1}}$ differs from the actual B_{n-1} by an amount of ϵ_{n-1}, then

$$\overline{B_{n-1}} = B_{n-1} - \epsilon_{n-1} . \tag{3}$$

If $\vec{\gamma}^\circ$ and $\overrightarrow{\Delta V}_n^\circ$ denote the unit vectors along the vectors $\partial\vec{B}/\partial V$ and $\overrightarrow{\overline{\Delta V_n}}$ respectively, then the equation (2) can be written:

$$\left|\frac{\partial \vec{B}}{\partial V}\right| \left|\overrightarrow{\overline{\Delta V_n}}\right| \cos(\vec{\gamma}^\circ , \overrightarrow{\Delta V}_n^\circ) + \overline{B_{n-1}} = 0. \tag{4}$$

The minimum of $\overline{\Delta V_n}$ requires the condition

$$\cos(\vec{\gamma}^\circ , \overrightarrow{\Delta V}_n^\circ) = 1 \tag{5}$$

i.e., the velocity-sensitivity vector $\partial\vec{B}/\partial V$ and the velocity correction vector $\overrightarrow{\overline{\Delta V_n}}$ are parallel. They have the same direction if $\overline{B_{n-1}} < 0$ and opposite direction if $\overline{B_{n-1}} > 0$ according as the passage of the spacecraft is at right or left of Jupiter. The computed velocity correction is therefore

[1]For a detailed and complete analysis of the spacing-ratio method, see ref. [3].

$$|\overline{\Delta V}_n| = \frac{|\overline{B_{n-1}}|}{\left|\frac{\partial B_n}{\partial V}\right|}. \tag{6}$$

Conditions (1) and (5) leads us to conclude the following optimal constant-attitude criterion:

The constant-attitude thrust optimization leads to constant-attitude velocity-sensitivity.

If the magnitude of the actual mechanized velocity correction is

$$\Delta V_n = \overline{\Delta V}_n + V'_n, \tag{7}$$

where V'_n is the velocity mechanized error, then the expression of the miss distance after nth correction becomes

$$B_n = B_{n-1} + (\overline{\Delta V}_n + V'_n)\,\frac{\partial B_n}{\partial V}. \tag{8}$$

Taking into account of the equations (2) and (3), the condition (8) becomes

$$B_n = \epsilon_{B_{n-1}} + V'_n\,\frac{\partial B_n}{\partial V}. \tag{9}$$

From equations (6), (7), and (9) we obtain

$$\Delta V_n = \frac{\overline{B_{n-1}}}{\frac{\partial B_n}{\partial V}} + V'_n. \tag{10}$$

Replacing n by n - 1 in equation (9) and using equations (3) and (10) we now obtain

$$\Delta V_n = V'_n + \left[\frac{\partial B_n}{\partial V}\right]^{-1}\left(\epsilon_{B_{n-2}} - \epsilon_{B_{n-1}} - V'_{n-1}\,\frac{\partial B_{n-1}}{\partial V}\right). \tag{11}$$

Equation (11) gives us the expression for the nth velocity correction and shows its connection to errors at and prior to the nth junction point.

Now the main concern is, given t_N and an RMS launching error σ_{B_0}, choose the correction or junction points $C_1, C_2, \ldots, C_N$, Figure 2, as to achieve the required mission objectives with a minimum total velocity correction and hence a minimum fuel expenditure for corrective thrusts. In other words, given t_N and the standard deviation σ_{B_0}, choose a sequence of times $\{t_n;\ n = 1, 2, \ldots, N\}$, with the integer N not specified such that the sum of the magnitude of the velocity corrections

$$\sum_{n=1}^{N} |\Delta V_n| \tag{12}$$

or more workable and closely related expression

$$S_N = \sum_{n=1}^{N} E[|\Delta V_n|] \tag{13}$$

is as small as possible. N denotes the total number of corrective thrusts, and $|\Delta V_n|$ is the magnitude of the impulse applied at time t_n. The symbol $E[|\Delta V_n|]$ indicates the mean value or expected value of the random variable $|\Delta V_n|$.

If C is a time average of the effective exhaust velocity, the relation between the fuel requirement of the total number of thrusts and the total velocity correction is given by:

$$\sum_{n=1}^{N} \Delta V_n \cong C \, Ln \, \lambda, \tag{14}$$

where λ denotes the mass ratio of the initial propellant mass to the final. The condition (14) holds for all mass ratios up to an exhaust velocity of one-tenth the velocity of light.

ANALYTICAL TREATMENT

The algebraic miss distance B at the target planet Jupiter is a function of time, position and velocity which remains constant between impulses. It is convenient to suppose that the miss distance is positive if the spacecraft passes to the left of the target planet and negative if the spacecraft passes to the right of the target planet.

The control-effectiveness or the velocity-sensitivity vector $\partial\vec{B}/\partial V$ is to be evaluated along the nominal trajectory and its magnitude is generally a monotonic decreasing function of time.

Before analyzing the sum S_N, given by equation (13), let us make two assumptions about the main error distributions.

The first assumption is that the distribution of thrust mechanization error is supposed to be normal with zero mean and standard deviation σ.

The second assumption is that the RMS velocity correction uncertainty in the direction of $\partial\vec{B}/\partial V$ is negligible.

Bearing in mind the above assumptions, the sum S_N can be written [3] as follows:

$$S_N = \left(\frac{2}{\pi}\right)^{1/2} \left\{ \left[\sigma^2 + \frac{\sigma^2_{B_0}}{\left(\frac{\partial B_1}{\partial V}\right)^2}\right]^{1/2} + \sigma \sum_{n=2}^{N} \left[1 + \left(\frac{\frac{\partial B_{n-1}}{\partial V}}{\frac{\partial B_n}{\partial V}}\right)^2\right]^{1/2} \right\} \tag{15}$$

$$= \left(\frac{2}{\pi}\right)^{1/2} \sigma F,$$

where

$$F = \left[1 + \frac{\sigma^2_{B_0}}{\sigma^2 \left(\frac{\partial B_1}{\partial V}\right)^2}\right]^{1/2} + \sum_{n=2}^{N} \left[1 + \left(\frac{\frac{\partial B_{n-1}}{\partial V}}{\frac{\partial B_n}{\partial V}}\right)^2\right]^{1/2} . \tag{16}$$

$B_0 = B(t_0)$ is the initial miss distance, which is a random variable, and supposed to be normally distributed with zero mean and standard deviation σ_{B_0}.

The first term of (15) corresponds to the case $n = 1$ and the second term is valid for $n > 1$.

The miss distance $|B_n|$ decreases after each correction and $|B_n| \to 0$, on the average, as $t_n \to T$, the arrival time. In other words, to an upper bound on the expected values of the last $|B_n|$ corresponds an upper bound on the time to target at the last correction, so that

$$t_N \geq t^x , \tag{17}$$

where t^x denotes the earliest time for the final correction.

Since $|\partial\vec{B}/\partial V|$ decreases to zero, the inequality (17) leads us to the following inequality

$$\frac{\partial B_N}{\partial V} \leq \frac{\partial B^x}{\partial V}, \quad B^x = B(t^x) , \tag{18}$$

which gives an upper bound on $\partial B_N/\partial V$.

Minimization of S_N, or the optimum spacing of correction times $\{t_n \; ; \; n = 1, 2, \ldots, N\}$ occurs [3] when

$$\left.\begin{aligned} &\frac{\partial B_N}{\partial V} = \frac{\partial B^x}{\partial V} \\ &\frac{\dfrac{\partial B_1}{\partial V}}{\dfrac{\partial B_2}{\partial V}} = \frac{\dfrac{\partial B_2}{\partial V}}{\dfrac{\partial B_3}{\partial V}} = \cdots\cdots = \frac{\dfrac{\partial B_{N-1}}{\partial V}}{\dfrac{\partial B_N}{\partial V}} = \rho \end{aligned}\right\} \tag{19}$$

and that if

$$\sigma_{B_0} > \rho\,\sigma\,\frac{\partial B_0}{\partial V} \tag{20}$$

then $t_1 = 0$, while if

$$\sigma_{B_0} < \rho\,\sigma\,\frac{\partial B_0}{\partial V} \tag{21}$$

then t_1 is obtained by

$$\frac{\partial B_1}{\partial V} = \frac{\sigma_{B_0}}{\rho \sigma} \; . \tag{22}$$

Condition $t_1 = 0$ states that if the launching error is sufficiently large in comparison with subsequent errors, the first correction to the spacecraft trajectory should be made in the vicinity of the earth, i.e., several days after launch so that one can get adequate early determination of miss distance.

The criterion for choosing $\partial B_N/\partial V$, which appears in (19), is [3]:

$$\frac{\partial B_N}{\partial V} = 1.6 \, \frac{\mu}{V_R^3} \left(1 + \frac{R V_R^2}{\mu}\right)^2 \; . \tag{23}$$

Where R denotes the nearest approach to the Jupiter's center, Figure 3, V_R is velocity of approach relative to the Jupiter, and μ is Gauss' constant for Jupiter.

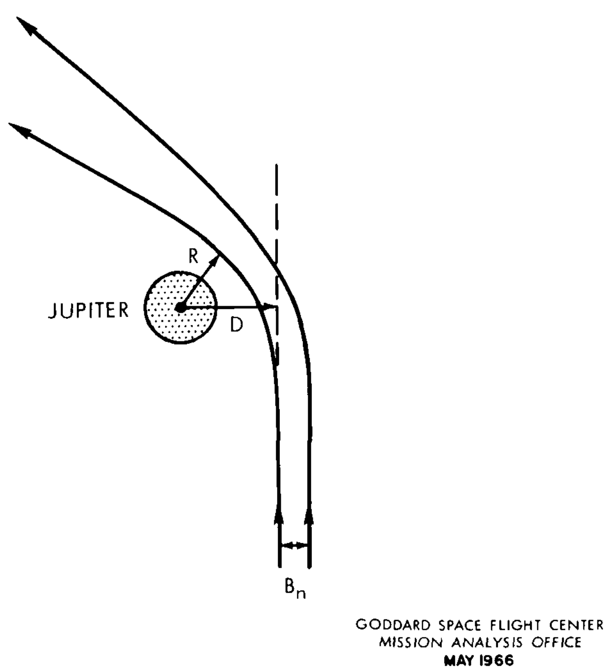

Figure 3–Trajectory Near Jupiter (Hyperbola)

The total number of corrective thrusts N is given [3] by

$$N = \begin{cases} 1 + \log_\rho \left(\dfrac{\dfrac{\partial B_0}{\partial V}}{\dfrac{\partial B_N}{\partial V}} \right) \quad \text{(nearest integer), if } \sigma_{B_0} > \rho\sigma \dfrac{\partial B_0}{\partial V} \\ \\ \log_\rho \left(\dfrac{\sigma_{B_0}}{\sigma \dfrac{\partial B_N}{\partial V}} \right), \text{ if } \sigma_{B_0} < \rho\sigma \dfrac{\partial B_0}{\partial V} \end{cases} \tag{24}$$

The minimized F is

$$F = \begin{cases} \left[1 + \dfrac{\sigma^2_{B_0}}{\sigma^2 \left(\dfrac{\partial B_0}{\partial V} \right)^2} \right]^{1/2} + (1 + \rho^2)^{1/2} \log_\rho \left(\dfrac{\dfrac{\partial B_0}{\partial V}}{\dfrac{\partial B_N}{\partial V}} \right) \\ \\ (1 + \rho^2)^{1/2} \log_\rho \left(\dfrac{\sigma_{B_0}}{\sigma \dfrac{\partial B_N}{\partial V}} \right) \end{cases} \tag{25}$$

Actually the number of midcourse maneuvers in a time-open mission depends on many factors such as the characteristics of the nominal trajectory and objectives of mission, and so on. It can be seen from equation (24) that for a given nominal trajectory and an RMS launching error σ_{B_0}, the total number of midcourse maneuver N, though limited, increases with t_N.

NUMERICAL ANALYSIS AND RESULTS

The nominal trajectory used for this galactic probe is adopted from [7] and has the following elements and characteristics:

Heliocentric Eccentricity	$e = 0.8655$
Heliocentric Semi-major axis	$a = 1.09318 \times 18^9$ km

Heliocentric Semi-minor axis	$b = 0.54758 \times 10^9$ km
Heliocentric Semi-Latus rectum	$\ell = 0.27429 \times 10^9$ km
Launch date	December 30, 1969
Injection time	$7^h\ 5^m\ 37^s$ GMT
Geocentric injection speed	15.405 km/sec
Flight time to Jupiter	500 days
$\mu_{sun} = Gm_{sun}$	$\mu_s = 0.132715115 \times 10^{12}$ km^3/sec^2

The trajectory near the planet Jupiter (hyperbola) has the following characteristics:

Eccentricity	$e_1 = 2.479$
Semitransverse	$a_1 = -0.7077 \times 10^6$ km
Radius of closest approach to Jupiter	$R = 10^6$ km
Velocity of approach relative to the planet Jupiter	$V_R = 20.8$ km/sec
$\mu_{jupiter} = Gm_{jupiter}$	$\mu_j = 0.12671059 \times 10^9$ km^3/sec^2.

In order to verify either of the inequalities (20), (21), and solve the equation (24), the numerical values of the standard deviations σ_{B_0}, σ, as well as the value of $\partial B_N / \partial V$, for a specified t_N, must be determined.

Taking into account of the following numerical data [7]:

$$\sigma_{B_0} \cong 2.4 \times 10^6 \text{ km}$$

$$\frac{\partial B_0}{\partial V} \cong 0.518 \times 10^5 \text{ km/m/sec}$$

$$\sigma \cong 0.5 \times 10^{-3} \text{ km/sec},$$

one finds that the inequality (20) is satisfied, and therefore the first form for N in equation (24) should be adopted.

In order to get an estimate for the upper bound of t_N one may refer to the guidance chart for Earth-Jupiter transfer [8] in Figure 4, which represents the variation of the control-effectiveness $|\partial\vec{B}/\partial V|$ along the above mentioned nominal trajectory.

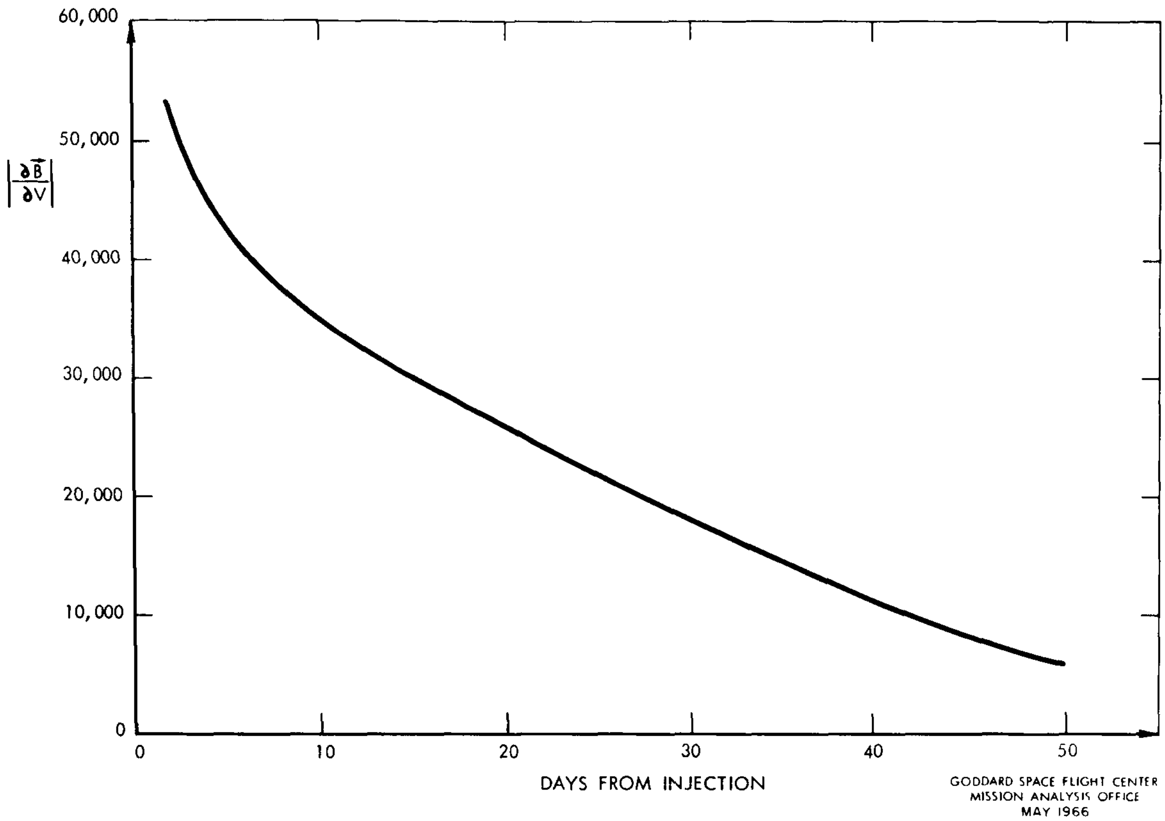

Figure 4–Guidance Chart for Earth-Jupiter Transfer

From Figure 4 it can be seen that the approximate time for the upper bound of t_N is 62 days from injection, and if we take for

$$t_N = 50 \text{ days}$$

after injection the optimum spacing ratio is

$$\rho \cong 2.5 \tag{26}$$

On the other hand

$$\frac{\partial B(50)}{\partial V} = 5500 \text{ km/m/sec},$$

therefore the optimum integer N, given by the first form of equation (24) is

$$N = 3. \tag{27}$$

The sequence of times $\{t_n\}$ for which the condition (17) is satisfied will be

$$t_1 = 10 \text{ days}, \; t_2 = 35 \text{ days}, \; t_3 = 50 \text{ days}. \tag{28}$$

Figure 4 shows that $|\partial \overrightarrow{B(t)}/\partial V|$ is a monotonic decreasing function of time and the sequence

$$\frac{\partial B_1}{\partial V} = 35000 \text{ km/m/sec.}, \; \frac{\partial B_2}{\partial V} = 14400 \text{ km/m/sec.}, \; \frac{\partial B_3}{\partial V} = 5500 \text{ km/m/sec.} \tag{29}$$

is geometric with the approximate ratio 1/2.50.

A glance at the literature shows that the optimum spacing ratio varies practically in the range

$$2.1 < \rho \leq 3 \tag{30}$$

so that the value obtained for ρ is not substantially different from those obtained by others.

The determination of fuel requirement for the total velocity correction as well as the magnitude of each velocity correction is of the utmost interest. In fact, the fuel expenditure is connected with the minimization of S_N or F which is given by upper form of equation (25):

$$F = \left[1 + \frac{\sigma^2_{B_0}}{\sigma^2 \left(\frac{\partial B_0}{\partial V}\right)^2}\right]^{1/2} + 2.939 \text{ Ln} \left(\frac{\frac{\partial B_0}{\partial V}}{\frac{\partial B_N}{\partial V}}\right), \tag{31}$$

where

$$(1 + \rho^2)^{1/2}/\text{Ln } \rho = 2.93857 \text{ for } \rho = 2.50.$$

As it can be seen from the above condition the first velocity correction ΔV_1, depends on the first term in Eq. (31), while the remaining corrections depend on logarithmic term. A detailed investigation of this problem should follow later.

CONCLUSIONS

The application of the spacing-ratio method to the case of a fly-by jovian mission leads us to the following conclusions:

1. The optimum choice for the spacing of corrections is one which minimizes the total velocity correction or, equivalently, minimizes the total required fuel.
2. A constant spacing-ratio of $\rho \cong 2.5$ for the velocity-sensitivity leads to a near optimum spacing of correction times.
3. The total number of corrective thrusts obtained by this method is higher, and therefore it is advisable to use this method if higher accuracies and more specific mission objectives, such as entry to the atmosphere of a target planet, or soft landing, are required for the terminal trajectory.

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REFERENCES

1. Pfeiffer, C. G., "A Dynamic Programming Analysis of Multiple Guidance Corrections of a Trajectory," AIAA Journal, Vol. 3, 1674-1681, 1965.

2. Battin, R. H., "A Statistical Optimizing Navigation Procedure for Space Flight," ARS J. 32, 1681-1696, 1962.

3. Breakwell, J. V., "The Optimum Spacing of Corrective Thrusts in Interplanetary Navigation," Mathematics in Science and Engineering (Academic Press, Inc., New York, 1962), Vol. 5, Chapter 12.

4. Strieble, C. T., and Breakwell, J. V., "Minimum Effort Control in Interplanetary Guidance,: IAS Paper 63-80, January 21, 1963.

5. Lawden, D. F., "Optimal Programme for Correctional Maneuvers," Astronautica Acta VI, 195-205, 1960.

6. Vonbun, F. O., "Transfer Geometry, Communications Conditions and Definitions for Galactic Probe Analyses," GSFC, Mission Analysis Office Technical Study, October 4, 1965.

7. Groves, R. T., "Injection and Midcourse Correction Analysis for the Galactic Probe," GSFC, Mission Analysis Office Technical Study, January 24, 1966.

8. Groves, R. T., Private Communication.